# An Analytical Model For The Energy Hole Problem In Many-To-One Sensor Networks

Jian Li and Prasant Mohapatra
Department of Computer Science
University of California, Davis
Email: {lijian, prasant}@cs.ucdavis.edu

***Abstract*— In a many-to-one sensor network, all sensor nodes generate CBR data and send them to a single sink via multihop transmissions. Sensor nodes sitting around the sink need to relay more traffic and suffer much faster energy consumption rates (ECR), and thus have much shorter expected lifetime. This may result in severe consequences such as early dysfunction of the entire network. While this phenomenon was reported previously in the existing literature, there is a lack of an analytical model on the characteristics of this issue. In this paper we present a mathematical model and characterize the energy hole problem. Using our model, we investigate the effectiveness of some existing approaches towards mitigating this problem in a formal manner. We have used simulation results to validate our analysis.**

***Index Terms*— Analytical model, Energy hole problem, many-to-one sensor networks, uneven energy consumption rates.**

## I. Introduction

Sensor networks [1] can be divided into two major classes, namely, many-to-many and many-to-one networks. In a many-to-many network, traffic flows between random pairs of source and destination nodes. In a many-to-one network, traffic from all sensor nodes is directed to a *single* sink (basestation) for further processing. Many-to-one sensor networks have various applications such as data gathering, monitoring and surveillance [2], [3], [4].

Although in-network information processing and reasoning are expected to take important roles in large scale sensor networks (e.g. see [5], some form of central functionality such decision-making and action commanding is required in many applications, which normally depend on a many-to-one communication model. In this paper, we investigate the uneven energy consumption issue which is associated with the many-to-one communication model in sensor networks.

System lifetime of a sensor network has different definitions based on the desired functionality. It may be defined as the time till the first node dies. It may also be defined as the time till a proportion of nodes die. If the proportion of dead nodes exceeds a certain threshold, it may result in uncovered sub-regions, and/or network partitioning. The location of the failure nodes is also of importance. If the proportion of nodes that have run out of battery are located in some critical part of the network, e.g., connecting the central sink and the rest of the network, it may result in early dysfunction of the entire network. Although it is not our intention to give a formal definition of sensor network lifetime in this paper, our discussion in the rest of this paper should be taken in the spirit of the second "definition."

The organization of the rest of this paper is as follows. In Section II, we describe and analyze the "energy hole" problem. Using our model, effectiveness of some existing approaches is analyzed in Section III, followed by Section IV on simulation validation. Related work is discussed in Section V. The paper is then concluded in Section VI.

## II. The "Energy Hole" Problem

### A. Preliminaries

To facilitate the discussion, we make some reasonable assumptions as follows:

- Each node continuously generates constant bit rate (CBR) data ($b$ bits/sec) and sends to a common sink through multihop shortest routes.
- Nodes are uniformly and randomly distributed, so the node density is uniform throughout the network: $p = \frac{N}{A_{net}}$, where $N$ is the number of nodes and $A_{net}$ is the network area.
- Sensor nodes have the same transmission range of $r$ meters.
- Ideal MAC layer, i.e., there is no collision and restransmission.
- Initially the network is well connected.
- Each link always has enough capacity to transfer the data.

For the energy model, we consider power for sensing, power for receiving and power for transmitting. The energy consumption formulas we use throughout this paper are as follows:

$P_{Sense} = \alpha_1 b$,
$P_{Tx} = (\beta_1 + \beta_2 r^n) b$,
$P_{Rx} = \gamma_1 b$,

where $b$ (in bits/sec) is individual node's data rate. The term $r^n$ accounts for the path loss, and the typical value for $n$ is 2 or 4. According to [15], some typical values for the parameters are as follows:

$\alpha_1 = 60 \times 10^{-9} J/bit$,
$\beta_1 = 45 \times 10^{-9} J/bit$,
$\beta_2 = 10 \times 10^{-12} J/bit/m^2$ (when $n = 2$),
or, $\beta_2 = 0.001 \times 10^{-12} J/bit/m^4$ (when $n = 4$),
$\gamma_1 = 135 \times 10^{-9} J/bit$.

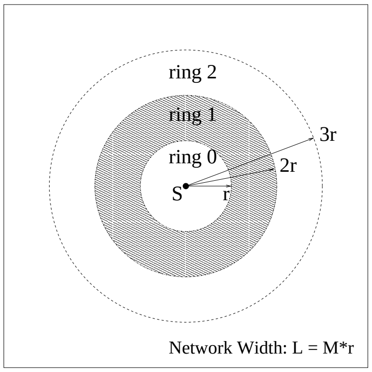


Fig. 1. The existence of "energy hole" around the sink node

### B. Description of the Problem

As illustrated in Figure 1, an $L \times L$ network with sink node S in the center, is divided into $M$ concentric bands. Note all traffic has to go through a node in ring 0. So, per node traffic load in ring 0 is:

$$Load_{ring\,0} = \frac{\text{total traffic in the network}}{\text{num of nodes in ring 0}} = \frac{p(Mr)^2 b}{p\pi r^2} = \frac{M^2}{\pi} b. \quad (1)$$

Similarly, we can obtain the per node traffic load in the other rings:

$$Load_{ring\,1} = \frac{\text{total traffic from outside ring 0}}{\text{num of nodes in ring 1}} = \frac{p\left((Mr)^2 - \pi r^2\right) b}{p\left(\pi(2r)^2 - \pi r^2\right)} = \frac{(\frac{M^2}{\pi} - 1)}{3} b, \quad (2)$$

More generally,

$$Load_{ring\,i^{th}} = \frac{p\left((Mr)^2 - \pi(ir)^2\right) b}{p\left(\pi\left((i+1)r\right)^2 - \pi(ir)^2\right)} = \frac{(\frac{M^2}{\pi} - i^2)}{2i+1} b. \quad (3)$$

We can observe there is considerable difference between the per node traffic load in different rings. Nodes in inner rings have much higher traffic burden, which we term as the "energy hole" problem.

### C. Characterization of the Model

The nodes in ring 0 have to relay the traffic from outer rings, in addition to sensing and transmitting its own data. The per node energy consuming rate (ECR) in ring 0 is:

$$ECR_{ring\,0} = \alpha_1 b + \gamma_1(\frac{M^2}{\pi} - 1)b + (\beta_1 + \beta_2 r^n)\frac{M^2}{\pi} b. \quad (4)$$

Similarly, we can derive the energy consumption rates in other rings:

$$ECR_{ring\,i^{th}} = \alpha_1 b + \gamma_1 \frac{\left(\frac{M^2}{\pi} - (i+1)^2\right)}{2i+1} b + (\beta_1 + \beta_2 r^n)\frac{(\frac{M^2}{\pi} - i^2)}{2i+1} b. \quad (5)$$

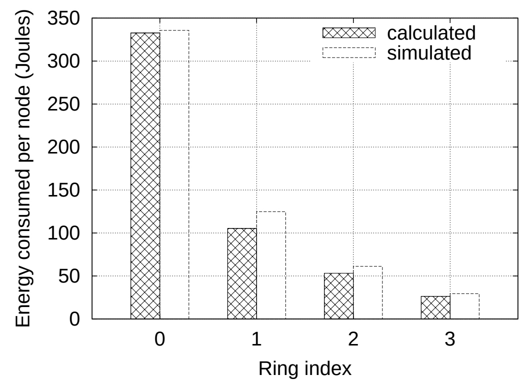


Fig. 2. Energy consumptions in different rings (when $M = 8$)

In order to verify our analytical model, we have done simulations with a 2000×2000 meters network. As shown in Figure 2, we observe that the simulated results match well with the analysis results. Nodes in inner rings have much higher energy consumption rates.

## III. Effectiveness of Different Approaches

In this section we use our model to investigate the effectiveness of several approaches in the existing literature towards mitigating the "energy hole" problem.

### A. Deployment Assistance

We observe that network density doesnot appear in the expression of per node energy consumption. From this, we can infer that simply deploying more nodes in the networks cannot prolong system lifetime. As shown in Figure 3, our simulation results confirms this.

Hence, researchers proposed clustering and sleep management techniques to prolong network lifetime. TTDD [14] attempts to use geographical grids to exploit high node densities to prolong system lifetime. In practice, we can deploy a bunch of assisting nodes with much higher (compared to normal sensor nodes) battery sources and a large transmission range, which form a relay layer on top of the normal sensors in the network. With assisting nodes as local sub-sinks, individual sub-regions are smaller, and the "energy hole" problem is expected to be alleviated.

## B. Traffic Compression and Aggregation

As the data packets are relayed from outer rings towards the sink in the center, each ring can exploit data redundancy and spatial correlation to aggregate and compress the traffic. Our work on a wavelet-based approach for time series compression and dissemination in sensor networks was summarized in [8]. Specific compression and aggregation techniques and related issues, such as data accuracy and error variance, are beyond the scope of this paper.

Let us assume that nodes in each ring can obtain a compression ratio $\alpha = \frac{\text{output load}}{\text{input load}} < 1.0$. (Note that in practice the compression ratio is highly related to the specific application and the routing scheme.) Since the network area consists of $m = \frac{M}{2}$ rings (plus four negligible corners), an approximation of per node load in ring 0 is:

$$\begin{aligned} Load_{ring\,0} &\approx \alpha^{m-1} D_{m-1} + \alpha^{m-2} D_{m-2} + ... + \alpha D_1 + b \\ &= b + \sum_{i=1}^{m-1} \alpha^i D_i, \end{aligned} \tag{6}$$

where

$$D_i = \frac{p\left(\pi(ir)^2 - \pi(ir-r)^2\right) b}{p\pi r^2} = (2i-1)b, \tag{7}$$

which is the per node relaying traffic that is generated from the $i^{th}$ ring and imposed on a single node in ring 0. So we get:

$$Load_{ring\,0} \approx b + \sum_{i=1}^{m-1} \alpha^i (2i-1) b. \tag{8}$$

Because of the fact that

$$\begin{aligned} & \sum_{i=1}^{t} \alpha^i (2i-1) \\ = \; & \sum_{i=1}^{t} 2(i+1)\alpha^i - 3\left(\sum_{i=1}^{t} \alpha^i\right) \\ = \; & \frac{\alpha + \alpha^2 - (2t+1)\alpha^{(t+1)} + (2t-1)\alpha^{(t+2)}}{(1-\alpha)^2} \\ \ll \; & t^2 + 2t \qquad \text{(when } t \geq 1\text{)}, \end{aligned} \tag{9}$$

we obtain:

$$\begin{aligned} Load_{ring\,0} &\approx b + \sum_{i=1}^{m-1} \alpha^i (2i-1) b \\ &\ll b + \left((m-1)^2 + 2(m-1)\right) b \\ &< \frac{M^2}{\pi} b, \end{aligned} \tag{10}$$

where the last term is the traffic load in ring 0 when there is no compression. That means, the relaying burden in ring 0 is greatly reduced.

**Remarks** In real applications, as the data packets transfer from outer rings towards the sink, they will be combined with locally-sensed data at each intermediate ring, and some form of compression will be applied to the aggregated data before it is forwarded to the next ring. Our approximation in above discussion is mathematically equivalent to the real process.

# IV. Simulation Validation

We have done extensive simulations to verify our analysis. In this section we present the simulation setups and results. In all the simulations we assume that MAC layer is ideal, i.e., there is no collision and retransmission which can result in extra energy consumption. Since our goal is to investigate the "energy hole" problem, we assume that each link always has enough capacity to transfer the data. If not stated otherwise, we use 250 meters as transmission range and $n = 4$ is chosen as the path loss factor. We run multiple simulations and obtain average results.

## A. Impact of Node Density

As observed from Equation (5), network density does not affect the energy consumption rates. To verify this feature, we have done simulations with a $2000 \times 2000$ meters network area. The number of nodes varies from 500, 600, 700, 1000, 1500 to 2000, which represents different node densities. For each node density, we run the simulation with different random seeds for multiple times. The bit rate is 2000 bits/second. Each run of simulation last for 2000 seconds. Finally we obtain the average results, as shown in Figure 3.

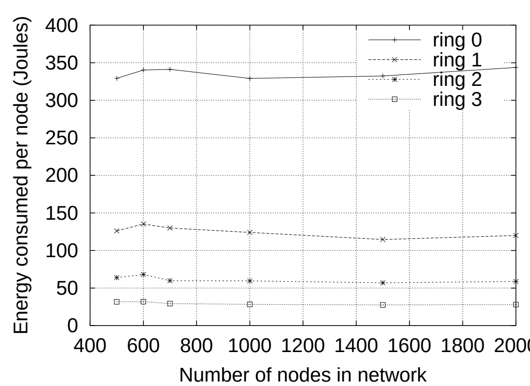


Fig. 3. Impact of different network node numbers

From Figure 3 we observe that, for each ring, the energy consumption stays at a steady level under different node densities. That is, per node energy consumption is independent of node density (assuming node density is adequate to guarantee network connectivity ), which justifies that we cannot prolong network lifetime by simply deploying more nodes.

### *B. Impact of Hierarchical Deployment*

To investigate the impact of hierarchical deployment, we run simulations with a 3000×3000 meters network. We use different division granularities by dividing the area into 1×1, 2×2, 3×3, and 4×4 grids. Correspondingly, the grid width is 3000, 1500, 1000, and 750 meters. Correspondingly, there are 0, 4, 9, and 16 assisting nodes, plus the single sink node.

We can observe in Figure 4 that, the energy consumption rate in ring 0 is greatly reduced, even with a 2×2 division method.

We would like to point out that the energy consumption in ring 0 (the case without division) in Figure 4 is much higher than that in Figure 3. This is because we use a larger network width (3000 meters), compared to 2000 meters that is used in simulations for Figure 3.

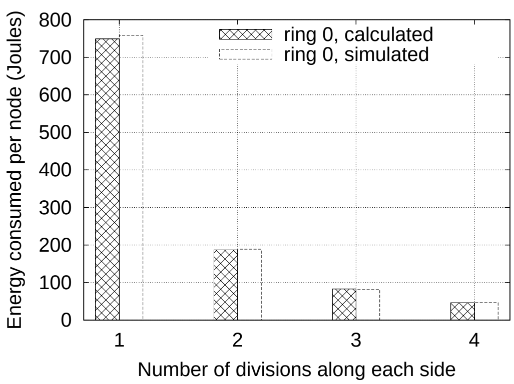


Fig. 4. Energy consumptions with sub-region division

### *C. Impact of Source Bit Rate*

From Equation (5), the energy consumption rate increases as the bit rate increases. To investigate the impact of source bit rate, we use a 2000×2000 meters network and vary the bit rates from 1000, 2000, 3000, to 4000 bits/second. For each bit rate, we run the simulation with different numbers of nodes from 500 to 2000. Each run of simulation last for 2000 seconds. The results are averaged over all runs of all scenarios.

Figure 5 shows the energy consumptions in different rings with varying bit rates. First, we can see that the simulated results match well with the analytical results. This validates the expression shown in Equation 4. Second, we observe that, as the bit rate increases, the energy consumption in ring 0 increases much faster than those in outer rings. This implies that, under the same

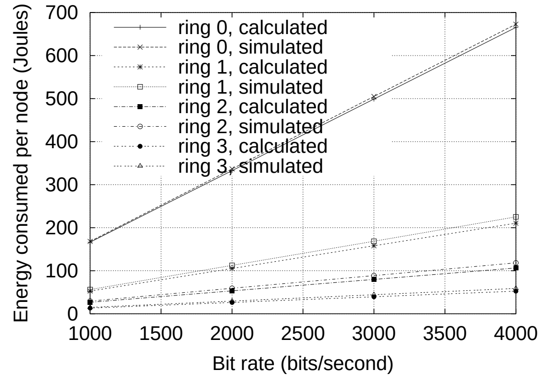


Fig. 5. Energy consumptions under different bit rates

network diameter, higher bit rates will make the "energy hole" problem even worse.

### *D. Impact of Traffic Compression*

In order to investigate the impact of traffic compression and aggregation, we use a 2000×2000 meters network with different node densities. The bit rate is 2000 bits/second and the packet size is 2000 bits. When a sensor node generates some data via sensing, or receives some data from other nodes, it will apply compression and aggregation technique to achieve a given compression ratio. We use 1.0, 0.9, 0.8, and 0.7 to represent different compression ratios, where a ratio equal to 1.0 means no compression is in use.

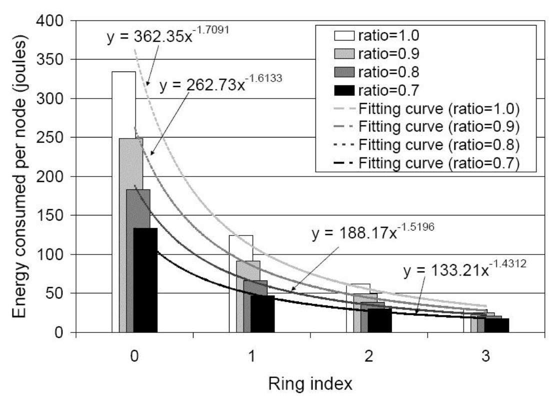


Fig. 6. Impact of traffic compression

The simulation results for different compression ratios are shown Figure 6. We observe that, as the compression ratio increases, the energy consumption rate in each ring decreases. As shown in the figure, we do curve fitting for each compression ratio case. We observe that, as the compression ratio is reduced from 1.0, to 0.9, 0.8, till 0.7, the power index of the fitting curve decreases from 1.7091, to 1.6133, 1.5196, till 1.4312. We can say that, the greater the compression degree, the flatter the fitting curve. In other words, the decrease in ring 0 is relatively greater than that in the outer rings, which helps even out the consumption rates in different rings.

## V. Related Work

In this section we briefly discuss the related work on bounding the fundamental limits, capacity and lifetime, in ad hoc and sensor networks. We also talk about some related work on deployment assisted approaches in this field.

### A. Bounding Lifetime

Bhardwaj et al have worked on upper bounds on the lifetime of sensor networks [6], [7]. In [6] the authors provided an analytical model for the lifetime issues based on trigger-based, many-to-one sensor networks. In [7], the authors further presented a role assignment technique in constructing the upper bounds on sensor network lifetime. To the best of our knowledge, these papers are among seminal efforts in this field. However, they do not identify the problem of uneven energy consumptions in many-to-one sensor networks.

In a more recent work [2], Duarte-Melo et al investigated extending sensor network lifetime by using hierarchical clustering technique. However, we use a totally different model for energy consumption analysis. Specifically, we identify the "energy hole" problem in the many-to-one traffic pattern.

### B. Bounding Capacity

A lot of work has been done on bounding the capacity of ad hoc networks and sensor networks [3], [9], [10]. All of these works assume that each node generates the same amount of data, i.e., the per node capacity is uniform throughout the entire network. Among them, [3] discussed the capacity issue in many-to-one sensor networks. According to our analysis that in many-to-one sensor networks those nodes close to the sink have to relay more traffic than others in outer rings, it would be a good idea to deploy more bandwidth capacity to inner sub-regions as needed. This type of structure aware capacity planning needs further research efforts.

### C. Deployment Assisted Approaches

Deployment assisted approaches have been previously proposed to improve the performance of ad hoc and sensor networks [11], [12], [13]. In [11], Ahmed et al proposed to deploy some assisting gateways in a mobile ad hoc network in order to provide better connectivity and facilitate scalability. Based on some assumptions they derived an approximate algorithm to compute the optimal positions where the gateways should be placed. In a more recent work [12], Ye et al proposed to deploy some reliable nodes in order to provide redundancy and better reliability in ad hoc routing protocols. The authors in [13] investigated the infrastructure tradeoff in sensor network deployment, which inspired the discussion aoubt the deployment assistance approach in this work.

## VI. Concluding Remarks

In this paper we develop an analytical model for the "energy hole" problem in many-to-one sensor networks. Based on the understanding of the characteristics of the "energy hole" model, we investigate the effectiveness of various existing techniques in mitigating this problem.

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