Probability Density of the Received Power in Mobile Networks

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***Abstract*—Probability density of the received power is well analyzed for wireless networks with static nodes. However, most of the present days networks are mobile and not much exploration has been done on statistical analysis of the received power for mobile networks in particular, for the network with random moving patterns. In this paper, we derive probability density of the received power for mobile networks with random mobility models. We consider the power received at an access point from a particular mobile node. Two mobility models are considered: Random Direction (RD) model and Random way-point (RWP) model. Wireless channel is assumed to have a small scale fading of Rayleigh distribution and path loss exponent of 4. 3D, 2D and 1D deployment of nodes are considered. Our findings show that the probability density of the received power for RD mobility models for all the three deployment topologies are weighted confluent hypergeometric functions. In case of RWP mobility models, the received power probability density for all the three deployment topologies are linear combinations of confluent hypergeometric functions. The analytical results are validated through NS2 simulations and a reasonably good match is found between analytical and simulation results.**

***Index Terms*—Probability density of received power, RWP mobility, RD mobility, centralised network.**

I. Introduction

Received power and its distributions have been used as signature for security schemes, signal, and interference analysis in the static networks [1], [2]. In case of Rayleigh fading channel, the received power probability distribution was found to be exponential function for static networks [2]. However, mobility is an inherent property of wireless networks and there is an increasing interest in exploiting mobility as a mechanism to enhance security and capacity [3], [4]. One of the fundamental characteristics of mobile networks is the time varying nature of the received signal strength, which is attributed to mobility, multipath fading and path loss via distance attenuation. As a result of this phenomenon, the received signal strength at the central access point changes over time at multiple time scales. The impact of such a signal variation affects not only the physical layer signal strength analysis but also other layer protocols such as routing, medium access, and security. In such mobile networks, designing algorithms (e.g., security algorithms, routing algorithms or interference calculations) based solely on received power (or signal strength) may not be optimal as the position of the node keeps changing and hence the channel characteristics. Therefore, it is appropriate to work with the statistics of the received power in the mobile networks as they change quite slowly in comparison to the signal itself. The statistics of the received signal has not yet been analyzed thoroughly in mobile networks, especially when the moving pattern is random. In this paper, we obtain a closed form expression for the probability density of the received power in mobile networks with random moving models. Some of the applications of the probability density of the received power are: (a) It can be used in Bayes risk and hypothesis testing to establish trust in the network [5]. In addition, it can be used as a signature to identify the genuine node in mobile networks using likelihood ratio statistical test [6]. (b) Probability density of the received power is also useful in estimating the interference level in the network [2]. It can also be used in determining probability of error and probability of correct detection in systems as explained in the Appendix of this paper. (c) Estimates of the received power can be used in hand off, power control, and channel assignment algorithms which helps in increasing the system capacity in cellular networks [7].

We make the following assumptions in this paper:

1) Wireless channel is considered to have Rayleigh fading.
2) A centralized access point surrounded by a set of nodes in 802.11 based mobile network is assumed. Users transmit/receive packets to/from the centralized access point using unique MAC address. This model can also be applied to cellular networks or centralized communication networks where each node transmits/receives packets to/from the central base station using an unique Pseudo random code.
3) The network topology is spherical with unit radius for 3D networks, circular with unit radius for 2D networks. The access point is assumed to be at center for both 3D and 2D topologies. For 1D topology, we consider a line with unit length and access point at the origin of the line. For theoretical analysis each node is assumed to transmit at equal and unit power.
4) Two mobility models, the Random Direction (RD) and Random way-point (RWP) models, are considered for illustration. However, the proposed approach is generic and can be extended to other mobility models as well.

II. Related Work

In [8], a distribution for received power is obtained by combining the Pareto distribution due to unknown locations with log normal shadowing. Probability density function (PDF) of the power received by an arbitrary antenna in a reverberation chamber is shown as an exponential distribution for a static network in [9]. The probability distribution of the received power in a static adhoc network with log normal shadowing and distance dependent path loss model is derived in [10].

Manuscript received December 20, 2010; revised June 8, 2011 and August 16, 2011; accepted August 18, 2011. The associate editor coordinating the review of this paper and approving it for publication was D. Tarchi.

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Digital Object Identifier 10.1109/TWC.2011.11.102250

Statistics of the received signal power at the central base station in a cellular direct-sequence code-division multiple-access (DS-CDMA) system with Rayleigh flat fading is studied in [11]. Probability density of the indoor received signal strength (RSS) for a WLAN is obtained in [12]. The distribution of the received power is shown to be a Rayleigh function for the nodes moving with a constant speed in [13]. To the best of our knowledge, there is no work on the received power statistics/distributions in a mobile network for different mobility models and topologies.

III. Signal Model

Let us assume that signal x is transmitted from a given mobile node to the central access point in a Rayleigh fading channel. The PDF of the received amplitude is given by

$$f_x(x) = \frac{x}{\sigma^2} exp\left(\frac{-x^2}{2\sigma^2}\right),\ x \geq 0 \tag{1}$$

where $2\sigma^2$ is the average received power of the signal. We assume that the mean power level falls of as forth power of range [14], [15]. Hence,

$$2\sigma^2 = \frac{P_T}{d^4} = \frac{1}{d^4} \tag{2}$$

where P_T represents multiplications of transmitted signal power, transmitter-receiver antenna gains and system loss, d is the distance between the access point and the given node. For simplicity, in (2) we have assumed $P_T = 1$. In case of static node distribution, $2\sigma^2$ is a constant. Now, the probability density of the received power at the access point from a static node which is at distance d is given by [16]

$$f_r(r) = \frac{1}{2\sigma^2} exp\big(\frac{-r}{2\sigma^2}\big),\ r \geq 0 \tag{3}$$

From (2) and (3), the PDF of the received power can be written as

$$f_r(r) = d^4 \exp\big(-rd^4\big),\ r \geq 0 \tag{4}$$

However, in case of mobile networks, the distance (d) between the access point and node itself changes randomly according to the mobility models. In the following section, we obtain the PDF of the received power for two different mobility models in a Rayleigh fading channel.

In random way-point mobility model a node is randomly positioned in a given space. An uniform random generator chooses the coordinates of a destination point. Node moves with a constant speed v to this destination then randomly chooses a next new destination (uniformly distributed), and so on. The speed v is randomly (uniformly) chosen from the interval $[v_{min},\ v_{max}]$. The time that the node takes to move from a starting position to its next destination is denoted as one movement *epoch*. Without loss of generality, we set the pause time in the destination point to zero. In case of random direction model a new direction is randomly (uniformly) chosen from $[0, \ldots, 2\pi]$ after a random movement epoch time, rather than a destination point. Further details of these two mobility models can be found in [17], [18] respectively.

IV. Probability Density of the Received Power with RD Mobility

In this section, first we assume that the nodes are moving as per Random Direction (RD) mobility model [18] in an unit space of 1D topology. The access point is at origin of a line and nodes are moving in the line over the space $[0,\ 1)$. Unit space is considered for simplification. Since the nodes have random mobility pattern, the distance d of a particular node from the access point is a random variable. Let us denote the PDF of distance d as $f_d(d)$. The steady state spatial distribution of the nodes in RD model follows uniform distribution over the space [18]. Now, the probability density of the position of a particular node is given by: $f_x(x) = 1,\ 0 \leq x \leq 1$. Since it is one dimensional movement, the distance d also has a similar distribution i.e., $f_d(d) = 1,\ 0 \leq d \leq 1$. Using simple mathematical calculations, we can obtain the PDF of the fourth order of the distance (i.e., PDF of $m = d^4$) as follows

$$f_m(m) = \frac{1}{4m^{3/4}},\ 0 \leq m \leq 1 \tag{5}$$

Since m is assumed to be deterministic constant in (4) (static network), the PDF of the received power for a given m can be written as

$$f_{r|m}(r|m) = m \times exp(-rm),\ r \geq 0 \tag{6}$$

Unlike static networks in mobile networks, m itself is a random variable. Therefore, from (5) and (6) the PDF of the received power can now be written as

$$\begin{aligned} P(r) &= \int_{m=0}^{1} f_{r|m}(r|m) f_m(m) dm \\ &= \int_0^1 \Big(m \times exp(-rm)\Big) \frac{1}{4m^{3/4}} dm,\ r \geq 0 \end{aligned} \tag{7}$$

The right hand side term in (7) can be expressed as a lower incomplete Gamma function as follows

$$\begin{aligned} \frac{1}{4}\int_0^1 m^{1/4} exp(-rm) dm &= \frac{1}{4}\frac{1}{r^{5/4}} \int_0^r t^{1/4} exp(-t) dt \\ &= \frac{1}{4}\frac{1}{r^{5/4}} \gamma\Big(\frac{5}{4}, r\Big) \end{aligned} \tag{8}$$

where $\gamma(.)$ is a lower incomplete gamma function given by $\gamma(s,r) = \int_0^r t^{s-1} e^{-t} dt$. From (7) and (8), we can write the PDF of the received power as

$$P(r) = \frac{1}{4}\frac{1}{r^{5/4}} \gamma\Big(\frac{5}{4}, r\Big),\ r \geq 0 \tag{9}$$

To simplify this further and to get a closed form expression, we can write the incomplete gamma function in terms of confluent hypergeometric function as follows [19]

$$\gamma(a, x) = a^{-1} x^a G_1(a, 1+a, -x) \tag{10}$$

where $G_1(.)$ is the confluent hypergeometric function of the first kind given by [19],

$$G_1(a,b,z) = 1 + \frac{a}{b}z + \frac{a(a+1)}{b(b+1)}\frac{z^2}{2!} + \ldots = \sum_{k=0}^{\infty} \frac{(a)_k}{(b)_k}\frac{z^k}{k!} \tag{11}$$

In (11) $(a)_n$ is rising factorial/Pochhammer symbol given by $(a)_n = a(a+1)(a+2)\ldots(a+n-1)$. Using (9) and (10), we can write

$$P(r) = \frac{1}{4r^{5/4}}\gamma\Big(\frac{5}{4}, r\Big) = \frac{1}{5}G_1\Big(\frac{5}{4}, \frac{9}{4}, -r\Big),\ r \geq 0 \quad (12)$$

This is a weighted confluent hypergeometric function. Now, the CDF can be derived as follows:

$$F_r(x) = \int_0^x P(r)dr = \frac{1}{5}\int_0^x G_1\Big(\frac{5}{4}, \frac{9}{4}, -r\Big)dr,\ r \geq 0 \quad (13)$$

Since, r is real and positive, we can write [19]

$$G_1(a, 1+a, -r) = e^{-r}G_1(1, 1+a, r) \quad (14)$$

From (11) and (14) we can write

$$\begin{aligned}\int_0^x G_1\Big(\frac{5}{4}, \frac{9}{4}, -r\Big)dr &= \int_0^x exp(-r)\sum_{k=0}^{\infty}\frac{(1)_k}{(\frac{9}{4})_k}\frac{r^k}{k!}dr \\ &= \int_0^x \sum_{k=0}^{\infty}\Big(\frac{(1)_k}{(\frac{9}{4})_k}\Big)\Big(exp(-r)\frac{r^k}{k!}\Big)dr \\ &= \sum_{k=0}^{\infty}\frac{(1)_k}{(\frac{9}{4})_k}\Big(\frac{1}{k!}\Big)\gamma(k+1, x)\end{aligned} \quad (15)$$

Now, from (13) and (15) we can get CDF as

$$F_r(x) = \int_0^x P(r)dr = \frac{1}{5}\sum_{k=0}^{\infty}\frac{(1)_k}{(\frac{9}{4})_k}\Big(\frac{1}{k!}\Big)\gamma(k+1, x) \quad (16)$$

We can get moment generating function of the received power by using the approach similar to (15)

$$\begin{aligned}M(t) = E[exp(tr)] &= \int_0^{\infty} exp(tr)P(r)dr \\ &= \Big(\frac{1}{5}\Big)\sum_{k=0}^{\infty}\Big(\Big(\frac{1}{(1-t)^{(k+1)}}\Big)\frac{(1)_k}{(\frac{9}{4})_k}\Big)\end{aligned} \quad (17)$$

Now, the mean of the received signal power is $M'(0) = \frac{d}{dt}(M(t))_{t=0} = \Big(\frac{1}{5}\Big)\sum_{k=0}^{\infty}\Big((k+1)\frac{(1)_k}{(\frac{9}{4})_k}\Big)$. We can see that this summation does not converge. Further we can verify that the other order moments do not converge either. It means that this probability distribution does not have either mean or other higher order moments. Cauchy distribution is one example case for this class of distribution.

By following a similar procedure, and employing the corresponding distance distributions we can obtain PDFs of received power for 2D and 3D deployments as well.

V. Probability Density of the Received Power with RWP Mobility

In this section we assume that the nodes are moving as per RWP mobility model [17], [20].

1D deployment of nodes

The PDF of distance d of a particular node from the access point in 1D RWP model over the unit space $[0,\ 1)$ is given by [17]

$$f_d(d) = -6d^2 + 6d,\ 0 \leq d \leq 1 \quad (18)$$

Now, the PDF of $m = d^4$ can be written as

$$f_m(m) = \frac{1}{2}\Big(\frac{-3}{m^{1/4}} + \frac{3}{m^{1/2}}\Big),\ 0 \leq m \leq 1 \quad (19)$$

By using an approach similar to (7), the PDF of the received power can be written as

$$\begin{aligned}P(r) &= \int_0^1 \Big(m \times exp(-rm)\Big)\frac{1}{2}\Big(\frac{-3}{m^{1/4}} + \frac{3}{m^{1/2}}\Big)dm \\ &= \frac{3}{2}\Big(-\int_0^1 m^{3/4}exp(-rm)dm \\ &\qquad + \int_0^1 m^{1/2}exp(-rm)dm\Big),\ r \geq 0\end{aligned} \quad (20)$$

By using a similar approach as in (8) and (12) we can get

$$P(r) = \frac{3}{2}\Big[-\frac{4}{7}G_1\Big(\frac{7}{4}, \frac{11}{4}, -r\Big) + \frac{2}{3}G_1\Big(\frac{3}{2}, \frac{5}{2}, -r\Big)\Big],\ r \geq 0 \quad (21)$$

This is a weighted sum of confluent hypergeometric functions. Using the approach similar to (15) we can derive the CDF as $F_r(x) = \frac{3}{2}\Bigg[\Big(\frac{-4}{7}\Big)\sum_{k=0}^{\infty}\Big(\frac{(1)_k}{(\frac{11}{4})_k}\Big)\Big(\frac{1}{k!}\Big)\gamma(k+1, x) + \Big(\frac{2}{3}\Big)\sum_{k=0}^{\infty}\Big(\frac{(1)_k}{(\frac{5}{2})_k}\Big)\Big(\frac{1}{k!}\Big)\gamma(k+1, x)\Bigg]$.

Now, the moment generating function of the received power is

$$\begin{aligned}M(t) = E[exp(tr)] = \Big(\frac{3}{2}\Big)\Bigg[&-\Big(\frac{4}{7}\Big)\sum_{k=0}^{\infty}\Big(\Big(\frac{1}{(1-t)^{(k+1)}}\Big)\frac{(1)_k}{(\frac{11}{4})_k}\Big) \\ &+ \Big(\frac{2}{3}\Big)\sum_{k=0}^{\infty}\Big(\Big(\frac{1}{(1-t)^{(k+1)}}\Big)\frac{(1)_k}{(\frac{5}{2})_k}\Big)\Bigg]\end{aligned} \quad (22)$$

Using (22) the mean of the received signal can be written as

$$\begin{aligned}M'(0) = \frac{d}{dt}(M(t))_{t=0} = \Big(\frac{3}{2}\Big)\Bigg[&-\Big(\frac{4}{7}\Big)\sum_{k=0}^{\infty}\Big((k+1)\frac{(1)_k}{(\frac{11}{4})_k}\Big) \\ &+ \Big(\frac{2}{3}\Big)\sum_{k=0}^{\infty}\Big((k+1)\frac{(1)_k}{(\frac{5}{2})_k}\Big)\Bigg]\end{aligned} \quad (23)$$

We can verify that this distribution has no mean as (23) does not converge. Similarly, the other order moments are not defined for this class of distribution either.

The distributions of distance d in RWP mobility for 2D and 3D deployments are given in [17], [20], [21]. Using these distance distributions and also same procedure as in (12) we can obtain the received power distributions for 2D and 3D deployments in RWP model. The results obtained are given in Table I where θ and ϕ are spatial coordinates.

From the analytical results shown in Table I, we can infer that the PDF of the received power is mobility-model and deployment topology dependent. The PDF plot of RD mobility model obtained from the theoretical derivation is provided in Fig. 1. From this figure we can observe that the PDF corresponding to 3D deployment dominates other two PDFs around the origin. Similarly, the PDF corresponding to 2D deployment dominates the 1D PDF around the origin. The

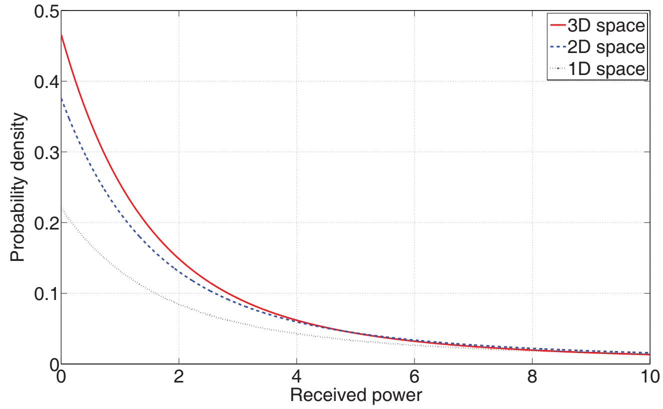

Fig. 1. Theoretical probability density function of received power in RD mobility model.

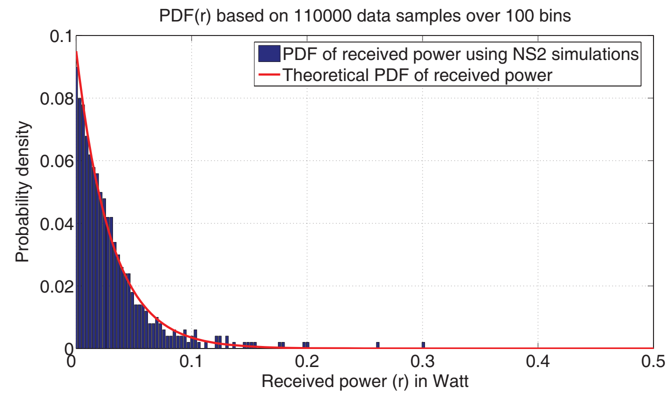

Fig. 2. Probability density function of received power in 1D RD mobility model.

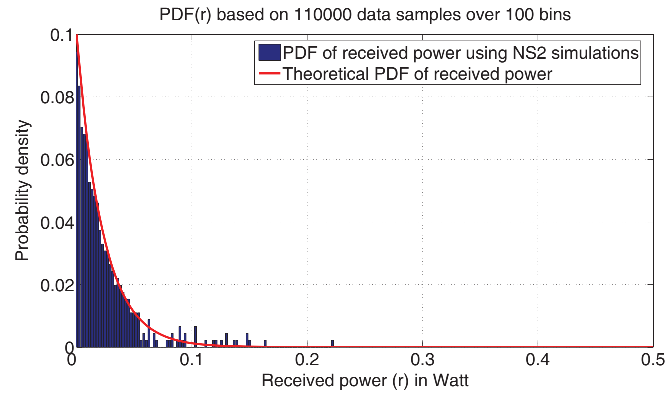

Fig. 3. Probability density function of received power in 1D RWP mobility model.

reason is that the spatial distribution of nodes corresponding to 3D deployment will have more weight far from the center than 2D and similarly, 2D will have more weight far from center than 1D for both RD and RWP mobility models. In 3D, since more nodes are far away from the center, received power will be less for most of the nodes (due to distance decay phenomenon) than 2D distribution. Therefore, the probability of receiving less power from any given node is higher for 3D distribution than 2D. Similarly, in 2D distribution, received power will be less for most of the nodes than 1D. Similar plot can be obtained for RWP mobility models also by using the expressions in Table I.

VI. Probability Density Functions using NS2 Simulations

We carried out the investigations in version 2.33 of NS2 by using the ad-hoc networking extensions provided by [22]. NS2 is used because it supports both RD and RWP mobility and also it has well defined wireless network models. We have implemented Rayleigh channel model using [23] by setting the Ricean K factor to 0. Simulation setup has one non-mobile access point and 10 mobile nodes moving in the space. Deployment space is spherical with $100m$ radius for 3D topology, circular with $100m$ radius for 2D topology and line of $100m$ length for 1D topology. The transmission range of each node is set at $110m$ so that all nodes can connect to the central access point in a single hop. The speed of the way-point is picked up uniformly random from the interval $[1,\ 5]m/sec$. IEEE 802.11b model is implemented with the UDP CBR (Constant Bit Rate) traffic of packet size 1024 and maximum number of packet is 110000. The transmit power is set as default value of $28\ mW$ in CMU model [22]. We measured the total received power (Pr_{tot}), which is returned by the propagation model [23]. Histogram over 100 bins is used to obtain PDF from the simulation trace file. The results and inference are provided in the following sections.

A. Probability density in RD and RWP mobility models

In Fig. 2 and Fig. 3, we have shown the results obtained using NS2 simulations and the theoretical results for the similar setup in 1D deployment of RD and RWP mobility models, respectively. The theoretical results are obtained by setting the transmit power to $28\ mW$ instead of unit power and by using the expressions in Table I. From these graphs, we can see that the probability density using simulations considerably match with mathematical derivation. The reason for the slight mismatch is that the theoretical derivation is for ideal setup and over infinite time duration. However, the simulation samples are limited in number and also have some random effects as per the simulation settings and the inherent limitations in the NS2 environment. Similar graphs can be obtained for 2D and 3D deployments.

B. Comparison of the probability density functions

We have plotted PDFs of received power in 2D deployment and 3D deployment in Fig. 4 and Fig. 5, respectively. PDFs for RD mobility model, RWP mobility model and static node distribution are compared for both 2D and 3D deployments. In static deployment, the node is assumed to be static at a distance of $50m$ from the access point. From Fig. 4 and Fig. 5, we can see that the PDFs differ from one another for different mobility models. Hence, these density functions can be used in the likelihood ratio test or other statistical tests to identify the intruders.

VII. Conclusion and Discussions

We have derived closed form expressions for the received power PDFs for both RD and RWP mobility models consider-

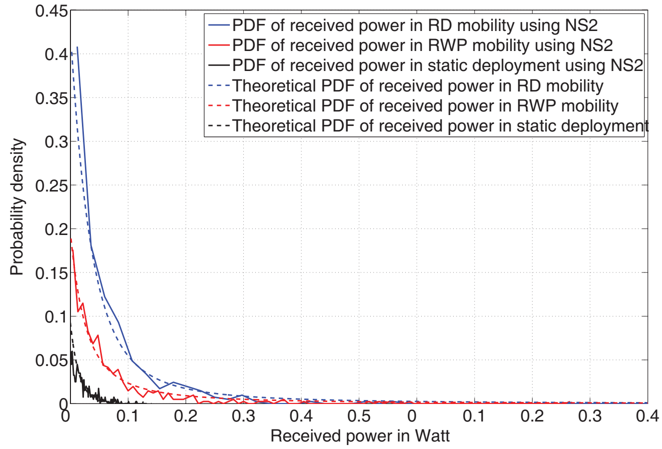

Fig. 4. Probability density functions of received power in a 2D node deployment for various mobility models (in static deployment the node is located at 50m from the center).

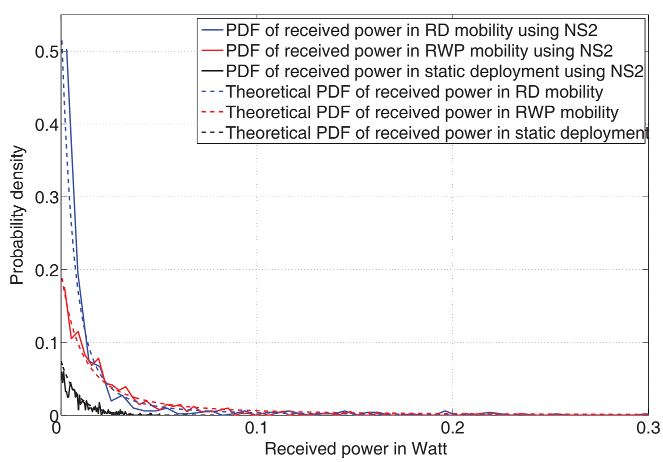

Fig. 5. Probability density functions of received power in a 3D node deployment for various mobility models (in static deployment the node is located at 50m from the center).

ing Rayleigh fading. Some of the statistical properties of the received power distribution such as moments and CDF have been anaylsed. The derived PDF also has been cross verified numerically using NS2 simulations. This theoretical PDF derived in this paper can be employed in several applications. We have demonstrated one application of the derived PDF. We have the following remarks about our approach:

- This approach is based on the steady state distance distributions obtained over infinitely larger duration derived in [17], [20]. These steady state distributions are independent of speed, time instant and channel correlation values. These distributions can closely match and serve as a good approximation for the nodes which stay in the network for long time. The initial distributions of the nodes' location/distance are different than the steady state distributions. However, after certain duration of time, the distributions of the received power will be unchanged and will be same as that of the steady state distributions.
- We have considered path loss exponent of 4. This model works well when $d > h_t + h_r$ where h_t and h_r are the transmitter and receiver antenna heights. However, considering the base station height, receiver antenna height and coverage area in cellular networks, the number of nodes satisfying $d < h_t + h_r$ will not be significantly large. The same argument can be applied to 802.11 wireless networks with mobile nodes as well. For the nodes which are closer to the base station/access point, the path loss model of equation (2) has to be redefined with the appropriate distance decay components. The rest of the procedures and derivations are same.
- In a typical mobile radio propagation situation, the received signal will have fading consisting of very rapid fluctuations around the mean signal level superimposed on relatively slow variations of the mean level. The fast variations are caused by multipath propagation, due to this the amplitude distribution of the signal is generally modelled as Rayleigh distribution. The slow mean signal variations are caused by shadow effects which is usually found to be lognormally distributed. In theoretical work, the mean value is generally assumed to be constant for simplification. This theoretical analysis can help us to get an approximate performance evaluation and performance bounds for the class of applications listed in the introduction section of this paper.

Appendix
Application of the Probability Density of the Received Power

A. *Noise alone systems*

In this section, we determine the probability of correct detection and hence the probability of error using the probability density of the received power obtained in this paper. For illustration purpose, we assume a mobile network in 1D deployment and all the nodes are moving as per the RD mobility model. First, we consider a noise alone system where the symbol will be detected successfully as long as $SNR > \Theta$. Now, the probability of correct detection is given by

$$\begin{aligned} p_c &= P(SNR > \Theta) = P(\frac{r}{\eta} > \Theta) \\ &= \int_{\Theta\eta}^{\infty}\int_0^1 \Big(m \times exp(-rm)\Big)\frac{1}{4m^{3/4}}dmdr \\ &= \frac{1}{4\sqrt{\Theta\eta}}\gamma\Big(\frac{1}{4},\Theta\eta\Big) \end{aligned} \tag{24}$$

where η is the noise power.

B. *Noise and Interference systems*

In the interference and noise system, the probability of correct detection is given by $p_c = P(SINR > \theta) = P(r > \theta(I+\eta))$ where $SINR$ is the signal to interference and noise ratio and θ is some threshold, I is the interference in the network and η is the noise power. Now

$$\begin{aligned} p_c &= E_I\left[\int_0^1\left(\frac{1}{4m^{3/4}}exp\Big(-\theta(I+\eta)m\Big)\right)dm\right] \\ &= \int_0^\infty\cdots\int_0^\infty\left[\int_0^1\left(\frac{1}{4m^{3/4}}exp\Big(-\theta(\sum_{i=1}^{k}\hat{r}_i+\eta)m\Big)\right)dm\right] \\ &\qquad\times\prod_{i=1}^{k}P(\hat{r}_i)d\hat{r}_1\ldots d\hat{r}_k \end{aligned} \tag{25}$$

where $\hat{r}_i$ is the interference power received from the ith interferer, k is the total number of interferers in the network and $E_I[.]$ is the statistical expectation operator with respect to I. Without loss of generality we can assume that the interferences are from static node in the network. Therefore, the probability density of $\hat{r}_i$ is given by

$$P(\hat{r}_i) = \frac{1}{\hat{R}_i}exp(-\hat{r}_i/\hat{R}_i) \tag{26}$$

where $\hat{R}_i = \frac{p_i}{d_i^4}$ is the average received power from interferer i which is static at distance d_i and p_i is the transmitted power of interferer i. If we assume that the interferers are mutually independent, then by using (25) and (26) and by changing the order of integrations, we get

$$p_c = \int_0^1\left(\frac{1}{4m^{3/4}}exp\Big(-\theta\eta m\Big)\right).\left(\prod_{i=1}^{k}\frac{1}{1+m\theta\frac{p_i}{d_i^4}}\right)dm \tag{27}$$

Now, using partial fractions we can write

$$\begin{aligned} p_c &= \frac{1}{4}\int_0^1\frac{e^{-\theta\eta m}}{m^{3/4}}\sum_{j=1}^{k}\left(\prod_{\substack{i=1\\i\neq j}}^{k}\frac{1}{1-\frac{p_i d_j^4}{p_j d_i^4}}\right)\frac{1}{1+\frac{mp_j\theta}{d_j^4}}dm \\ &= \frac{1}{4}\sum_{j=1}^{k}\left(\prod_{\substack{i=1\\i\neq j}}^{k}\frac{1}{1-\frac{p_i d_j^4}{p_j d_i^4}}\right)\int_0^1\frac{e^{-\theta\eta m}}{m^{3/4}}\frac{1}{1+\frac{mp_j\theta}{d_j^4}}dm \end{aligned} \tag{28}$$

By making the substitution $m = y^4$ we get

$$p_c = \sum_{j=1}^{k}\left(\prod_{\substack{i=1\\i\neq j}}^{k}\frac{1}{1-\frac{p_i d_j^4}{p_j d_i^4}}\right)\int_0^1\frac{e^{-\theta\eta y^4}}{\left(1+\frac{y^4 p_j\theta}{d_j^4}\right)}dy \tag{29}$$

By using the Simpson's Quadratic interpolation rule for numerical integration, we can obtain the following approximation:

$$p_c \approx \sum_{j=1}^{k}\left(\prod_{\substack{i=1\\i\neq j}}^{k}\frac{1}{1-\frac{p_i d_j^4}{p_j d_i^4}}\right)\frac{1}{6}\left[1+4\frac{e^{-(\theta\eta/4)}}{\left(1+\frac{p_j\theta}{4d_j^4}\right)}+\frac{e^{-\theta\eta}}{\left(1+\frac{p_j\theta}{d_j^4}\right)}\right] \tag{30}$$

(30) will give estimate of the probability of correct detection and probability of error $(1-p_c)$ in the network. This analysis will hold good even when the interferers are mobile. In this case the $P(\hat{r}_i)$ in (26) will have a different distribution based on the mobility pattern.

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TABLE I
SUMMARY OF THE RECEIVED POWER PDF RESULTS

Mobility model	Space	PDF of distance $f_d(d)$	PDF of received power $f_r(r)$	Moment generating functions M(t)
RD	1D	$1,\ 0 \le d \le 1$	$\frac{1}{5}G_1\left(\frac{5}{4},\frac{9}{4},-r\right),\ r \ge 0$	$\frac{1}{5}\sum_{k=0}^{\infty}\left(\left(\frac{1}{(1-t)^{(k+1)}}\right)\frac{(1)_k}{(\frac{9}{4})_k}\right)$
	2D	$\frac{1}{\pi},\ 0 \le d \le 1,\ \ -\pi \le \theta \le \pi$	$\frac{1}{3}G_1\left(\frac{3}{2},\frac{5}{2},-r\right),\ r \ge 0$	$\frac{1}{3}\sum_{k=0}^{\infty}\left(\left(\frac{1}{(1-t)^{(k+1)}}\right)\frac{(1)_k}{(\frac{5}{2})_k}\right)$
	3D	$\frac{3}{4\pi},\ 0 \le d \le 1,$ $0 \le \theta \le \pi,\ 0 \le \phi \le 2\pi$	$\frac{3}{7}G_1\left(\frac{7}{4},\frac{11}{4},-r\right),\ r \ge 0$	$\frac{3}{7}\sum_{k=0}^{\infty}\left(\left(\frac{1}{(1-t)^{(k+1)}}\right)\frac{(1)_k}{(\frac{11}{4})_k}\right)$
RWP	1D	$-6d^2+6d,\ 0 \le d \le 1$	$\frac{3}{2}\left[-\frac{4}{7}G_1\left(\frac{7}{4},\frac{11}{4},-r\right)+\frac{2}{3}G_1\left(\frac{3}{2},\frac{5}{2},-r\right)\right],\ r \ge 0$	$\left(\frac{3}{2}\right)\left[-\left(\frac{4}{7}\right)\sum_{k=0}^{\infty}\left(\left(\frac{1}{(1-t)^{(k+1)}}\right)\frac{(1)_k}{(\frac{11}{4})_k}\right) + \left(\frac{2}{3}\right)\sum_{k=0}^{\infty}\left(\left(\frac{1}{(1-t)^{(k+1)}}\right)\frac{(1)_k}{(\frac{5}{2})_k}\right)\right]$
	2D	$\frac{12}{73}\left(27d-35d^3+8d^5\right),$ $0 \le d \le 1$	$\frac{3}{73}\left[27\left(\frac{2}{3}\right)G_1\left(\frac{3}{2},\frac{5}{2},-r\right)-35\left(\frac{1}{2}\right)G_1\left(2,3,-r\right) + 8\left(\frac{2}{5}\right)G_1\left(\frac{5}{2},\frac{7}{2},-r\right)\right],\ r \ge 0$	$\frac{3}{73}\left[27\left(\frac{2}{3}\right)\sum_{k=0}^{\infty}\left(\left(\frac{1}{(1-t)^{(k+1)}}\right)\frac{(1)_k}{(\frac{5}{2})_k}\right) - 35\left(\frac{1}{2}\right)\sum_{k=0}^{\infty}\left(\left(\frac{1}{(1-t)^{(k+1)}}\right)\frac{(1)_k}{(3)_k}\right) + 8\left(\frac{2}{5}\right)\sum_{k=0}^{\infty}\left(\left(\frac{1}{(1-t)^{(k+1)}}\right)\frac{(1)_k}{(\frac{7}{2})_k}\right)\right]$
	3D	$\frac{35}{72}d^2(21-34d^2+13d^4),$ $0 \le d \le 1$	$\frac{35}{288}\left[21\left(\frac{4}{7}\right)G_1\left(\frac{7}{4},\frac{11}{4},-r\right)-34\left(\frac{4}{9}\right)G_1\left(\frac{9}{4},\frac{13}{4},-r\right) + 13\left(\frac{4}{11}\right)G_1\left(\frac{11}{4},\frac{15}{4},-r\right)\right],\ r \ge 0$	$\frac{35}{288}\left[21\left(\frac{4}{7}\right)\sum_{k=0}^{\infty}\left(\left(\frac{1}{(1-t)^{(k+1)}}\right)\frac{(1)_k}{(\frac{11}{4})_k}\right) - 34\left(\frac{4}{9}\right)\sum_{k=0}^{\infty}\left(\left(\frac{1}{(1-t)^{(k+1)}}\right)\frac{(1)_k}{(\frac{13}{4})_k}\right) + 13\left(\frac{4}{11}\right)\sum_{k=0}^{\infty}\left(\left(\frac{1}{(1-t)^{(k+1)}}\right)\frac{(1)_k}{(\frac{15}{4})_k}\right)\right]$